Retention in Mathematics students: problems and possible approaches.

Penazzi, Davide

Retention in Mathematics students: problems and possible approaches.

D. Penazzi.

University of Central Lancashire, Preston, Lancashire, PR1 2HE UK

Abstract

This article describes and analyses the approach adopted by the Mathematics course team in UCLan to improve retention in the first year mathematics students. After introducing the key aspects of the skills required by a mathematics student and the teaching methods considered in the past to improve such skills, the UCLan method is outlined. Such method is based on a mixture of formative and summative assignments, spread throughout the year. A case study allows to statistically confirm the effectiveness of such method. We conclude the article outlining possible improvements and drawbacks.

Introduction

Starting university studies is a challenging step in the life of a student, who often struggles becoming more independent: “students are expected to accept personal responsibility for both academic and social aspects of their lives [; this] will create anxiety and distress, undermining their normal coping mechanisms” (Lowe, 2003).

Ensuring a proper development of the student and the accurate level of support is key in improving the retention rates, it is in fact demonstrated that the first year is the “make or break” year (Oldham 1988). Whilst in the days of the yore the idea of an elitist academia where only few could succeed was praised, in the modern days the focus is on giving everyone the opportunity to have a higher education.

A study conducted in Napier University (Johnston 1997) has shown that the main factor influencing the non-retention of students (identified by the course leaders) is “Academic Problems” (37%), followed by “Personal difficulties” (29%) and Financial Difficulties (12%). A lecturer, especially if lecturing first year modules, should be aware of this: whilst the psychological and financial support is mostly responsibility of the institution, it is fully the course team's duty to address student's academic problems.

Ideally a university student should be able, attending lectures, reading material and with some indication of what is expected, to independently organize one's time and prepare for the final exam. This is an unrealistic expectation on students just coming from sixth form; Entwistle (2000) observes that students during the course of their university years move from a surface learning (in which the focus is on repeating notions) to deep learning (where there is the use of the acquired knowledge to transform information and ideas). This is a necessary evolution of student's approach to learning, as surface learning is not sufficient (as it was in sixth form) to obtain a higher education degree. Entwistle argues that the shift to deep learning can occur “only where the assessment procedure emphasizes and rewards personal understanding”. In my personal experience this is certainly true, but the assessment only method will not suffice for most students: if in the student's education was only promoted surface learning, it is unlikely that the student will develop a more organized and deeper learning method by oneself. It is therefore fundamental that the lecturers foster the students and educate them on the learning strategies.

We are going to consider the main causes of failure in first year students, with a focus on the chiefly mathematical issues. It will then follow a review of the approaches tried in the past to improve retention, and a possible solution applied by the Mathematics course team in UCLan. We finish with a review of this pilot scheme with pros and cons.

Causes of failure in first-year mathematics students

A study of Van Etten identifies motivating factors for students. Obviously an enthusiastic lecturer is the main one, but, if we focus on the factors related to assignments, students observed that “If the deadline is in the distant future there is little motivation to do the assignment” (and thus if the course is assessed only with a final examination, students are prone to leave the lecture notes aside until a few days before the exam), “firm, clear deadlines motivate students to meet these deadlines” and “[students] self award themselves when they complete a task” (Van Etten 2008). So students seem to prefer regular assignments and tasks which help them organize their studies; such tasks moreover give them the 'award factor' of having completed a part of their examination. As “for many students poor performance is largely due to ignorance about the study skills required, or the inability to apply these skills appropriately, rather than lack of ability”, observes Anthony (2000), such regular assignments would teach the students the correct study skills.

Anthony’s article focuses on first year mathematics students; he observes that the main factor, identified by both students and lecturers, influencing success in the first-year is “self motivation”, followed by “study for tests and exams” (Anthony 2000). Surprisingly “assignment completion” is valued by students much more than it is by lecturers. Neither students nor lecturers considered the ability to think mathematically as important to pass exams; this is worrying as it might signify that mathematical thinking is not considered as a learning outcome and thus not properly assessed. In the following section I will explore mathematical thinking in more detail.

The main cause of failure identified by the students is “lack of effort”, followed by “lack of self motivation” (lecturers ranked these 2nd and 3rd respectively), the main cause of failure for lecturers was “insufficient work” (only 18th for the students). It appears then that to improve results the two chief things to work on is to increase students' motivation and make them put enough effort.

Studies of Kaiser and Wilson (2003) show that a key factor in students’ engagement (and thus performance) is the intention: a student who has the will to put effort and time in their studies will do it. The question to consider is then how can we foster intention? Eccles’ expectancy-value model (Eccles, 2009) say that a person will be more likely to engage in a challenging task if they know they can succeed and if they intrinsically value the task and its utility. Whilst the latter can be done in creating the right working environment and an adequate learning community; in this article we will really focus on the former: how to provide adequate tools to the first year students to successfully complete the year?

The mathematical context:

Some academic issues are specific to the study of mathematics and need to be considered carefully: the existence of threshold concepts, the fact that the new material builds up on material learned previously and the development of mathematical thinking.

Mathematical thinking is possibly the one thing which makes the study of mathematics unique, and which is extremely difficult to define. Fascinating arguments of (Burton 1984), (Devlin 2012) and (Kun 2013) show that mathematical thinking is not thinking about (or doing) mathematics, but a set of “operations, processes and dynamics of thinking”. The discipline of Mathematics is the discipline for which this way of thinking is the best approach; it is the mathematical thinking and not the mathematical notions learned during the university studies which are desired by employers and research institutes. The presence of mathematical thinking is hugely different between pre-18 mathematics taught in school (where the focus is on learning methods to solve standard problems) and higher education mathematics, where the focus should be on combining and synthesizing the mathematical methods and tools to solve a variety of new problems. At university level it can be observed that students with similar A-grades in maths obtain very different results depending on their being able to develop the mathematical thinking or sticking with the old standard methods, and it is thus necessary to find a way of check the students’ progress early in their career to break them of the habit of repeating standard procedures.

On the other hand mathematical thinking alone is not sufficient to obtain a Mathematics degree, there is a large amount of material which needs to be learned (the “tools of the trade”). This material, moreover, builds on material learned in previous modules (the commonly used analogy is that of topics as bricks to construct a wall: you need to have a good basis to build the next layer). Having obtained a deep understanding of the material in the first year is therefore especially important.

The last key feature is threshold concepts: they exist in any discipline, but in mathematics they appear continuously in a student's course of studies. Meyer defines them as follows: “A threshold concept [opens] a new and previously inaccessible way of thinking about something. It represents a transformed way of understanding, or interpreting, or viewing something without which the learner cannot progress.” (Meyer 2003) A threshold concept is “transformative” (produces a significant shift in the perception of a subject), “irreversible” (unlikely to be forgotten), “integrative” (exposing previously hidden interrelatedness of something) and “potentially troublesome” (Meyer 2003). Troublesome as overcoming a threshold concept forces the student to question his ideas and beliefs, reach a “liminal” (Meyer 2003) state in which the truth is hidden in a cloud of possibilities, and usually follows a trial and error phase after which the student 'clicks' and the new idea is acquired. This process takes different time with different students (and different threshold concept), also some students profit from help from the tutor or the peers, whilst others would rather work out the problem on their own. When designing a mathematics module the lecturer has to organize it to cater for all the diverse student body.

The aim:

Ideally the graduate of a mathematics course is a mathematician, someone with an “extended understanding” of mathematics, “being able to go beyond what has been taught, deal creatively with new situations” (Biggs 2003). This in contrast to someone who can solve algorithmically a limited set of mathematical problems; as computers can do this task, such individuals are of no interest to the job market. Being a mathematician implies not only knowledge of the subject but having overcome several thresholds in understanding (and thus having gained a certain mental flexibility necessary to deal with new problems), and being able to think mathematically. In the first year we can only start putting the foundation of this development, forcing the students out of the habit of just working through some examples to get a good grade. We have to apply constructive alignment (teaching method and assessment tasks aligned to the learning outcomes) (Biggs 2003) when designing our modules: assessment tasks must “mirror what you intended [the students] to learn” (Biggs 2002), in this case the ability to think mathematically and solve new problems.

It is certainly unfair to ask first year students to solve new problems in the short time of a final examination if they have only come across standard exercises throughout the year. “The good [students], the 'academic' ones, will themselves turn declarative into functioning knowledge in time but most will not if they are not required to” (Biggs 2003). The final-examination-only method would either yield a high failure rate, if the exam is set to really examine the learning outcomes, which contrasts with V2 of the UKPSF (“Promote participation in higher education and equality of opportunity for learners”) (UKPSF); or the exam would have to consist only of standard exercises, thus not assessing the correct development of the mathematician-to-be.

Some methods developed in the past:

Traditionally mathematics modules are taught only with plenary lectures and a final examination. Although it is widespread belief that due to the nature of the subject this is the best method to deliver mathematics modules (Di Leonardi 2007), it has proved good only for a minority of students, who already have an 'academic' attitude, and thus mathematics lecturers started devising alternative measures to support the lectures.

In a pamphlet produced by the HEA called “Mapping university mathematics practices” (Iannone 2012) it is described an assessment method (Chapter 13) which involves quick quizzes counting for 15% of the final grade and a presentation (5%) on top of the final year closed book examination (80%), to assess “students' continuous engagement with the material”. This also provides useful feedback to the students as if they score low in a quiz it “can bring them to a realization that they lack understanding of some very basic mathematical concepts”.

The assessment practice just described is also supported by a presentation by the students which is used to highlight misunderstanding of fundamental material. As Bezuidhenhout (Bezuidhenhout 2001) highlights “after diagnosing the nature of students' conceptual problems, [lecturers can] develop specific teaching strategies to address such problems and to enhance conceptual understanding”.

Although the pamphlet focuses on summative and not formative assessment, I believe that the very effective practice of regular quizzes or short class tests needs to be supported also by some form of formative assessment, as this provides students with the feedback on knowledge of the material and teaches them a study model, but does not help them with threshold concepts or tests their mathematical thinking. Furthermore the short time duration of the class tests does not encourage students to spend time on solving hard problems, but rather to have a “surface learning” approach, in which they study selectively what they assume might be examined (Entwistle, 2000).

We need students to engage in high level mathematical tasks, which “are often complex and longer in duration than more routine classroom activities” (Henningsen 1997), and thus practices like projects and presentations are needed to make students 'do mathematics'. In the first year the mathematical topics are too basic for any “research-tutored” activity, thus providing students with non-standard exercises and giving them the time to work could be the solution. Students need support with this activity, and tutorial classes in which the lecturer simply solves exercises at the board is of no help. During tutorial classes the lecturer needs to “make personal contact with the students, [to] clear up personal problems with the tutorial sheets”, and allow “problem solving by students to give knowledge, experience and confidence” (Searl 1979).

The pilot scheme at UCLan:

The Mathematics course team at UCLan has implemented a pilot scheme to improve students' performance and retention rates, it has been firstly tested on the modules MA1831 (Functions, Vectors and Calculus) in the year 2012/13 and then extended to the three pure mathematics first year modules: MA1811 Introduction to

Algebra and Linear Algebra, MA1812 Introduction to Real analysis and MA1831 in the year 2013/14. Having run the courses for two years there is now enough statistical data to analyse.

All the modules have the following structure: in each week there are three hours of frontal teaching: two hours of lectures and one hour of tutorial. There are ten class tests during the year, roughly fortnightly, which count for 30% of the final grade. These tests lasts 10 to 15 minutes and are done at the beginning of a lecture. Students are aware of the dates of these tests and the topics tested, so have time to prepare.

The final exam counts then for 70% of the final grade.

The scheme implemented involved weekly homework which students had to submit; these homework were then marked and returned after a week with feedback. They were not graded but students were given a “satisfactory” or “unsatisfactory” grade based on the “effort” taken in doing the homework, i.e. the homework was deemed satisfactory is students had given a serious attempt to more then 80% of the questions.

Such homework was compulsory although no penalty was given to students not submitting (or not submitting satisfactory work).

The tutorial session was devoted to help students complete the homework, returning the marked homework, giving general feedback and answering questions on topics students found difficult.

The homework allows students to work at their pace towards a variety of mathematical problems, where the mathematical thresholds are embedded, thus allowing students to have time to think, discuss them with the lecturer in tutorial sessions and with fellow students outside of the frontal teaching hours: developing mathematical thinking and overcome thresholds requires sufficient time and effort! Often in mathematical modules the entire material required by the student to be studied are one-hundred-pages lecture notes; which strike the new student for being a thin amount of material, but it is deceiving, as it extremely dense of information and requires dedication and commitment to be unravelled.

Compulsory homework force student to revise at short distance what has been explained, thus avoiding the false understanding so well described by a student interviewed by Orsini-Jones (2006): “I understood it in class, it was when I went away and I just seemed to have completely forgotten everything”. The “unravelling” effort is then done throughout the academic year, which considering the time constraints of exam sessions in UK, is the only possible way to pass the final year exam.

Moreover having these homework marked and returned to the students provides an effective method to give prompt feedback, and a good measure for the students to check one's understanding in preparation to the class tests and exams. Students who regularly produce homework will thus have evidence of their ability to understand and pass the module: one of the two key points of Eccles' expectancy-value model.

The module MA1831, unlike the other pure mathematics modules, is undertaken by first year students in Mathematics and first year students in Physics/Astrophysics. Whilst for the Mathematics students an entry level of B in Mathematics A-level is required, the requirement for the Physics cohort is C. This diversity is addressed in the choice of material, which builds only on the tools studied in GCSC Maths, but also needs to be considered when delivering lectures and setting the assignments. In fact Physics students do not necessarily know what is expected from them in a mathematics module and have possibly not experienced any instance of mathematical thinking in the past. There has been naturally a gap between the students in mathematics and in physics, the latter ones showing a high failure rate.

A study of Shaw (1997) considers the attitude of non-mathematics student (in their case engineering students) attending mathematics courses. They place students into five clusters: “High-flyers” (motivated and successful students), “Downhillers” (students with high starting expectations and that lose progressively interest in the subject obtaining poor results), “Haters” (students not motivated and not putting effort), “Realistics” (motivated students but with mediocre results due to the high workload) and “Ambivalents” (students not motivated but who put enough effort to obtain a decent grade). In my experience I did not come across the “Realistics” and “Ambivalents” but a different group of students which seemed interested but too lazy to obtain good results. A positive result in Shaw's study was that most students are motivated and want to improve their mathematical skills as they believe they could be useful in their subject. As in Anthony's study “both students and lecturers rated poor study techniques as a more influential factor in failure than inadequate mathematics background knowledge” (Anthony 2000), thus, with a push on regular work, we hope the scheme would reduce the gap between mathematics and physics students.

Statistical analysis

Submission of homework (and submitted homework ranked as satisfactory or unsatisfactory), and the results of class tests for the module MA1831: Functions, Vectors and Calculus have been considered. Students that withdrew or did not attend more than 65% class tests were excluded from the statistics, and the mean of the results of each class test has been considered separately for the mathematics and the physics cohorts.

In the academic year 2013/2014 the mathematics cohort regularly submitted the homework (each homework had 50% or more submission rate); whilst the physics cohort dropped their submission rate to less than 15% after class test 5.

The results were the following:

Class test	1	2	2	4	5	6	7	8	9	10
Maths mean	7.26	7.79	8.74	8.05	6.86	7.66	8.61	7.45	5.68	4.47
Physics mean	6.21	7.21	8.75	7.71	7	5.07	4.79	5.43	3.61	2.29

Table 1. Results of the class tests

On plotting them it is easy to observe a difference in the results of the two cohorts after Class test 5:

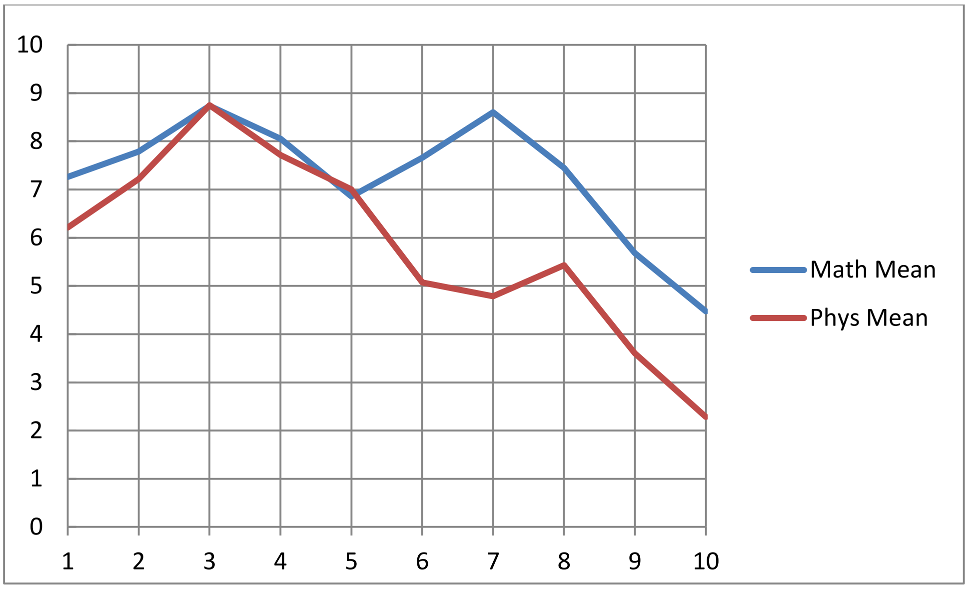

Figure 1. Plot of the means of the class tests.

In Figure 1 on the horizontal axis is the class test, on the vertical axis the mean mark.

It is thus worth asking if there is statistical significance showing that the lack of continuous homework production has caused a real drop in grades.

We used Mann-Whitney-Wilcoxon (MWW) test (Mann, 1947) to determine if the means are different; to justify the use of such test we observe that, due to the small numbers of the physics cohort and the nature of the class tests, it is not possible to assume that the marks follow a normal distribution; but it makes sense to assume both physics and mathematics cohorts follow the same distribution.

With null hypothesis “the two cohorts have the same mean mark in class test n” and alternative hypothesis “the mean mark of the maths cohort is higher than the physics one”, the MWW test shows (with $p<0.05$) that in class tests 2,3,4 and 5 both cohorts have the same mean mark, whilst in class tests 1,6,7,8,9,10 the maths cohort have better marks:

Class test	1	2	3	4	5
p-value	0.01709	0.3525	0.3201	0.2541	0.5084
Class test	6	7	8	9	10
p-value	0.002195	0.00002595	0.01358	0.0331	0.02195

Table 2. p-values of the MWW test.

To interpret this data first observe that the first class test could be not very significant, the physics cohort entry grades are slightly lower than the mathematics ones, thus physics students might not be prepared to a mathematics test at HE level.

This issue seems to be overcome by the second class test, and the two cohorts have similar results. The difference reappears after the physics cohort stopped doing the homework.

A further statistical analysis using MWW-test with null hypothesis "the mathematics cohort mean mark is 2 marks above the physics one" and alternative hypothesis its negation we can show that the drop can be quantified in two marks.

It might be argued that this is due to the material becoming harder, but in fact the topics of the first 5 class tests are familiar topics for students with an A-level in maths (vectors, functions, derivation and differentiation are, although in simplified form, part of the A-level curriculum), but the topics of the last 5 class tests are new to both cohorts, and thus the drop in physics marks is not justifiable in that way. In fact harder topics produced similar marks drops in both cohort and the difference of the means has remained similar for the latter 5 class tests.

At the exam the performance of Mathematics students was significantly better, with average mark of 61.1 in contrast with the Physics average mark of 47.7.

The retention rate in first year mathematics is particularly high: 91% (excluding failure due to external circumstances), despite the entry grades for the course are lower than in other universities with a similar degree. It is our belief that teaching students a working ethic and providing regular detailed feedback in the crucial transition year accounts for a large part of this success.

Possible criticism

One possible criticism to this scheme is that it does not allow students to develop study skills independently. This is certainly a major issue, a mathematics graduate is supposed to be able to organize one's own time, but, as, according to the UK report 'Measuring the Mathematics Problem': “This past decade has seen a serious decline in students' basic skills and level of preparation on entry into Higher Education” (Hawkes 2000), students are not taught sufficient study skills in secondary school, and thus is the duty of the university to teach them.

Therefore this scheme is not suitable for second and third year modules, the students have to get able to make their own weekly study plan. Second year modules should be then be transitional, with some mid-year form of assessment and with lecturers checking regularly that students are not slacking off during the year. Having studied throughout the first year, students might have found the preparation for the final examinations easy, and thus deduced that not much work is needed. In reality this is due to their regular effort and they need to be made aware of it.

Another major drawback of this method is that it requires considerable overheads; whilst the marking of the homework can be done externally, the class tests need to be marked by a lecturer, adding 5 hours per student to the normal yearly workload. Such scheme thus is really suitable only in a university where the cohort size is medium/small (less than 70 students) or at a significant extra cost.

This assessment strategy should not be thought as a stand-alone solution, but as part of a more holistic approach encompassing a variety of co-curricular support program activities, as highlighted by Davies and Hawwash (2013) and by Estrada in her pamphlet (Estrada). Some of the key issues in developing as a mathematician, in particular overcoming thresholds and the identification with the mathematical community are more suitably addressed with a mentoring scheme. As analysed by Rhodes and others (2006, p695) mentors can not only help students in the tasks but also to “youths’ positive identity development. That is, mentors may help shift youth’s conceptions of both their current and their future identity”). In UCLAN the mathematics team has assigned to each student a personal tutor, who takes the role of a mentor and to whom the students can refer to for help on top of the usual module leaders. Again such method has significant overheads, and a department with more students would probably prefer a student-to-student mentoring scheme.

Conclusions

The gap between school and higher education seems to continue to widen, and it is therefore necessary to develop strategies to support the transition to university and avoid large numbers of dropouts in the first year.

We have proposed a mixed formative/summative assessment practice for first year pure mathematics modules, aimed at teaching study skills for a mathematics course. We believe this can be a valuable contribution to practices in university across UK, in particular in universities with an intake of less than 70 students per year, where the method proposed does not pose significant overheads.

The data gathered supports the claim of effectiveness of this method, as long as it is used together with other co-curricular supports.

References:

Anthony, G.;(2000) Factors influencing first-year students' success in mathematics, International Journal of Mathematical Education in Science and Technology, 31:1,3-14.

Bezuidhenhout, J.;(2001) Limits and continuity: some conceptions of first-year students, International Journal of Mathematical Education in Science and Technology, 32:4, 487-500.

Biggs, J.; (2002) Constructive alignment in Action: Imaginative Curriculum Symposium, ITSN generic centre.

Biggs, J.;(2003) Aligning teaching and assessment to curriculum objectives, ITSN generic centre.

Burton, L.; (1984) Mathematical Thinking: The Struggle for Meaning, Journal for Research in Mathematics Education, Vol. 15, No. 1, Jan., 1984.

Davies, J. W., & Hawwash, K. (2013). Collecting and presenting guidance on transition, induction and retention of students in HE STEM. *Engineering Education*, *8*(1)

Devlin, K., blog “Devlin's Angle”, http://devlinsangle.blogspot.co.uk/2012/08/what-is-mathematical-thinking.html

Di Leonardi, BC;(2007), Tips for facilitating learning: the lecture deserves some respect, J Contin Educ Nurs. Jul-Aug;38(4):154-61.

Eccles, J.S.; (2009) Who am I and what am I going to do with my life? Personal and collective identities as motivators of action. Educational Psycologist, 44.

Entwistle, N.;(2000), Promoting deep learning through teaching and assessment: conceptual frameworks and educational contexts, paper presented at the TLRP conference, Leicester, November 2000.

Estrada, M.; Ingredients for improving the culture of STEM degree attainment with co-curricular supports for underrepresented minority students.

Hawkes, T., Savage, M. (eds.); (2000), Measuring the mathematics problem”, Report, the Learning and teaching support network, IMA, LMS, the engineering council.

Iannone, P., Simpson, A.; (2012), Mapping university mathematics assessment practices, HE stem.

Kaiser, F.G. and Wilson, M. (2004) Goal-directed conservation behaviour: The specific composition of a general performance. Personality and Individuality Differences, 36.

Kun, J., blog post “Mathematical thinking doesn’t look anything like mathematics”, http://j2kun.svbtle.com/mathematical-thinking-doesnt-look-like-mathematics.

Lowe, H., Cook, A.;(2003) Minding the Gap: are students prepared for higher education?, Journal of Further and Higher Education, Vol. 27, No. 1.

Mann, H., Whitney, D.;(1947) On a Test of Whether one of Two Random Variables is Stochastically Larger than the Other, Ann. Math. Statist., Volume 18, Number 1, 50-60.

Mayer, J., Land, R.;(2003), Threshold concepts and troublesome knowledge: linkages to ways of thinking and practising within the disciplines, Improving Student Learning – Ten years on, OCSLD, Oxford.

Oldham, B.;(1988), The first year - make or break year, Journal of Further and Higher Education, Vol 12, No 2, pp5-11.

Orsini-Jones, M.;(2006) "identifying troublesome concepts and helping undergraduates with crossing grammar thresholds via assessed collaborative group work". Threshold concepts within the disciplines Symposium. Glasgow. 2006.

Johnston, V.;(1997) Why do first year students fail to progress to their second year? An academic staff perspective. Paper presented at the British Educational Research Association Annual Conference September 11-14 1997: University of York

Rhodes et al.; (2006) A model for the influence of mentoring relationships on youth development, Journal of community psychology, 34.

Searl, J.;(1979) Tutorial classes in mathematics, International Journal of Mathematical Education in Science and Technology, 10:4, 553-555.

Shaw, C., Shaw, V.; (1997) First-year students' attitudes to mathematics, International Journal of Mathematical Education in Science and Technology, 28:2, 289-301.

UKPSF, http://www.heacademy.ac.uk/ukpsf.

Van Etten, S., et al.;(2008) College seniors' theory of their academic motivation, Journal of Educational Psychology, vol. 100, no. 4, 812-828.